Date: 24.03.2014
Teacher: Çiğdem Özdemir
Number of Students: 20
Grade Level: 10 IB-SL
Time Frame: 70

Mathematics Learning Plan: Exponential Equations

1. Goal(s)
   - Students will understand how to solve exponential equations.
   - Students will have an insight how they may encounter an exponential equation in their daily lives.

2A. Specific Objectives (measurable)
   - Students will be able to construct an exponential equation from a real life problem.
   - Students will be able to take logarithm of an equation.
   - Students will be able solve exponential equations.
   - Students will be able to apply their knowledge of law of logarithm into solving exponential equations.

2B. Ministry of National Education (MoNE) Objectives
   - Üstel ve logaritmik denklemlerin ve eşitsizliklerin çözüm kümelerini bulur.
   - Üstel ve logaritmik fonksiyonları gerçek/gerçekçi hayat durumlarını modelleme ve problem çözmede kullanır.

2C. NCTM-CCSS-IB or IGCSE Standards:
   - Students should understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions

3. Rationale
   - This lesson will enable students to realize that they need to know logarithm to solve real life problems such as population problems.

4. Materials
   - Computer,
   - Overhead projector,
   - PowerPoint presentation including the population problem.

- Worksheet

5. Resources
   - Oxford, IB Diploma Programme, Mathematics Standard Level
   - Hease Mathematics, Mathematics for the International Student, High Level
   - MEB, Ortaöğretim Matematik 11. Sınıf Ders Kitabı

6. Getting Ready for the Lesson (Preparation Information)
   - Teacher prepares name cards for students and writes students' name on those cards before the lesson.
   - Teacher prepares a checklist including student names.
   - Teacher prepares a Power Point presentation including the population problem.
   - Teacher prepares the attached worksheet.

7. Prior Background Knowledge (Prerequisite Skills)
   - Students should know the laws of logarithms.
   - Students should know change of base formula.
   - Students should know the relation between exponential and logarithmic functions.

Lesson Procedures

Transition: Teacher introduces herself and distributes students' name cards that she prepared before in order to get to know students well. Then teacher says "Today we are going to learn how to solve exponential equations"

8A. Engage (5)
   - Teacher asks students if they know about Turkey's annual population growth. Teacher takes students responses. Then she asks "What about Germany's annual population growth. Teacher talks with students about population change of these two countries. Then teacher asks students how Turkey's population would reach to Germany's population.

Transition: "Now let's see if Turkey's population can reach to Germany's population"

B. Explore (5)
   - Teacher reflects the information about population of Turkey and Germany on the board.
     "Population of Turkey was counted as 76 million people in 2013 and it was determined that population of people in Turkey is increasing in size by 1.3% per year. In the same year, population of Germany was counted as 82 million and it is seen that Germany's population is

stable. Teacher asks "With this information can you say that Turkey's population can reach to Germany's population" Teacher expects students to realize that Germany's rate of growth is less than Turkey's rate of growth.

- Teacher wants students to find out how many people will be in Turkey and Germany after 1 year later, 2 years later, and so on.
- If students need, teacher helps students to come up with the function of population by drawing a table.

Turkey		Germany	
Number of years	Population	Number of years	Population
0	76	0	82

- Teacher asks "If n years later the population will be the same, what should I do?"
- Teacher expects students to find functions which are like $76 * 1.013^n$ for Turkey and 82 for Germany.
- Teacher expects students to equalize these functions reminding that n years later the population will be same.

Transition: After students equalize these functions, teacher asks "how can I find n from this equation?"

C. Explain (30)

- Teacher explains that logarithm helps us finding the power of a function
- Teacher reminds how to take logarithm of a number.
- Teacher asks with which base I should take the logarithm. If students choose ten for the base, teacher goes on.
- After taking the logarithm both sides of the equation, teacher asks students the product rule of logarithm. She expects students to write $\log(76 * (1.013)\text{^}n) = \log 76 + \log(1.013)\text{^}n$
- The teacher then reminds how logarithm works with an exponential. She expects students to carry n to the front of log(1.013).
- Then teacher expects students to solve the equation by using their TI-84 calculators.
- After students find the answer, teacher asks students if their result is reasonable. Here, teacher expects students to verify their answer by finding each country's population n years later.

- After solving the question, teacher asks “What if I take the logarithm with base 5 or 6 or *e*. So teacher shows taking the natural logarithm of both sides.

Transition: “Now look at the examples in your book.”

D. Extend (15)

- Teacher assigns the exercise questions from the Oxford book.
- While students are solving the questions, teacher walks around and observes the common difficulties of students. She writes some of those questions that students have difficulties and solves them by reminding students the logarithm rules.

Transition: “Now let’s see what we have learnt”

E. Evaluate (10)

- Teacher distributes a worksheet which is covering the questions related to exponential equations and real life problems.

9. Closure & Relevance for Future Learning

- Teacher wants students to write their weaknesses and strengths related to this topic in a few words at the end of the lesson.

10. Specific Key Questions:

- What is an equation? (Knowledge)
- What is an exponential equation? (Synthesis)
- What are Turkey’s and Germany’s population? (Knowledge)
- Do you think Turkey’s population can reach to Germany’s population soon? (Analysis)
- To find out if Turkey’s population can reach to Germany’s population what do I need to know? (Analysis)
- After reflecting the population information of both countries on the board, teacher asks “With this information can I say that Turkey’s population can reach to Germany’s population soon?” (Analysis)
- How do you express the population of Turkey n years later? (Synthesis)
- What will Germany’s population n years later? (Comprehension)
- If I say n years later the population of both country will be same, what should I do? (Analysis)
- After students equalize the population of both countries n years later, teacher asks how I can find n from this equation. (Analysis)
- How were we taking the logarithm of an exponential? (Knowledge)

- With which base should I take the logarithm of both sides? (Comprehension)
- How logarithm works with an exponential? (Knowledge)
- Why I must take the logarithm of both sides? (Comprehension)
- After students find n, teacher asks "Is your answer reasonable?" (Evaluation)
- What if I take the logarithm with base 5 or 6 or *e?* (Application)
- What if Germany's population increases in size by 0.1 per year? (Analysis)
- How our equation would change? (Application)
- What was the log law for product? (Knowledge)
- What was the log law for division? (Knowledge)

11. Modifications
    - If students cannot find the function of Turkey's population n years later, teacher helps students finding it on the table.
    - If students do not remember the log laws, teacher reminds them.

Worksheet

1) Consider the equation $2^x = 30$.

   a Solve for x, giving an exact answer, by using: a base 2 b base 10.

2) Consider the equation $R = 200 \times 2^{0.25t}$.

   a Rearrange the equation to give t in terms of R.

   b Hence find t when: i $R = 600$ ii $R = 1425$

3) The weight of a radioactive isotope after t years is given by $W_t = 2500 \times 3^{-\frac{t}{3000}}$ grams.

a Find the initial weight of the isotope.

b Find the time taken for the isotope to reduce to 30% of its original weight.

4) The weight of a radioactive isotope after t years is given by $W_t = 2500 \times 3^{-\frac{t}{3000}}$ grams.

a Find the initial weight of the isotope.

b Find the time taken for the isotope to reduce to 30% of its original weight.

5) A population of seals is given by $P_t = P_0 2^{\frac{t}{3}}$ where t is the time in years, $t \geqslant 0$.

a Find the time required for the population to double in size.

b Find the percentage increase in population during the first 4 years.