

Date: 06.02.2014

Teacher: Çiğdem Özdemir

Number of Students: 15

Grade Level: 11 (HL- TM)

Time Frame: 90 minutes

Summation Notation

1. Goal(s)

- Students will understand the summation notation concept.
- Students will understand the applications of summation notation.

2A. Specific Objectives (measurable)

- Students will be able to write the finite sum using sigma notation.
- Students will be able to read sigma notation.
- Students will be able to evaluate a sum.

2B. Ministry of National Education (MoNE) Objectives

- Toplam sembolünü ve çarpım sembolünü açıklar, kullanışları ile ilgili özellikleri gösterir, temel toplam formüllerini modelleyerek inşa eder.

3. Rationale

- Students will understand how summation notation can help finding the sum of a wide range of consecutive numbers.

4. Materials

- Computer,
- Overhead projector,
- Power Point presentation including the word problem
- Two different color board markers.
- Worksheet and homework

5. Resources

- Turkish Education Association Publications, Mathematics 11, Hüseyin Aydın, Hilmi Selbes, M. Emin Özel.
- Tek Yıldız Matematik Modülü, www.tekyildizyayinlari.com

6. Getting Ready for the Lesson (Preparation Information)

- Teacher prepares name cards for students and writes students' name on those cards before lesson.
- Teacher prepares a checklist including student names.
- Teacher prepares a Power Point presentation including the word problem which is supposed to be solved in the class.
- Teacher prepares questions to be solved during the lesson.
- Teacher prepares the attached worksheet and homework.

7. Prior Background Knowledge (Prerequisite Skills)

- The students should know mathematical induction.
- The students should remember functions.
- The students should remember exponential functions.

Lesson Procedures

Transition: "As you have started the summation notation last week, today we will continue with the summation notation. Let's remember what summation notation was."

8A. Engage (5 minutes)

- Teacher asks the students to consider that one student has one Turkish lira in her pocket, and the student next to him/her has two Turkish liras in his/her pocket, and the student next to him/her has four Turkish liras in his/her pocket and so on, doubling the money next to him/her.
- Teacher asks the students how would they show the total money of the classroom by summation notation
- Teacher writes the money of each student on the board and asks the general term of this summation then reminds the students how to write the summation notation for this sum.

Transition: "Now let's consider this problem."

B. Explore (10 minutes)

- Teacher reflects a word problem about summation notation via PowerPoint:

“Sam is laying out patio stones in a triangular pattern. The first row has 2 stones and each row has 2 additional stones. How many complete rows can he make with a box of 144 stones?”

- Teacher gives about one minute students to think about the problem.
- Teacher asks students what they understood and make students understand the problem by asking questions such as “What are the important information given in this problem?”
- After thinking on the problem, teacher asks students to work on the problem with their peers for three minutes. Teacher walks around while students are working on the problem and take notes about students’ progress to her checklist.
- Teacher asks students about their solutions as she observed while students were working on the problems.
- After students work on the problem, teacher solves the problem on the board by asking questions such as “How can I draw the diagram of this problem”, “What is the pattern of this diagram?” “How can I express this pattern by summation notation?”

Transition: “Now let’s do further examples about summation notation by using properties of summation notation”

C. Explain (30 minutes)

- Teacher solves the questions below on the board
- Teacher reminds the properties that students had learned last lesson if needed.

1) Compute the sum

$$\sum_{k=5}^{40} \sqrt{2k-1} - \sqrt{2k+1}$$

2) Compute the sum

$$\sum_{k=1}^{180} \cos k$$

3) Compute the sum

$$\sum_{k=1}^{127} \log_2\left(1 + \frac{1}{k}\right)$$

- After solving the questions above, teacher informs students that not all the summation notation problems can be solved by expanding. Some of the problems require knowledge of sum formulas.
- Teacher reminds the sum formulas.
- Then teacher solves the questions below by using sum formulas and properties of summation notation.

4) Compute the sum

$$\sum_{k=1}^{10} k^2 - 5k - 20$$

5) Find the formula of

$$\sum_{k=1}^n k(k+1)$$

Transition: Let's solve some more questions about summation notation. I shall give you the worksheet and I want you to solve the questions with your desk mates.

D. Extend (30 minutes)

- The teacher distributes the worksheet papers to the students.
- Teacher wants students to check the properties of summation notation on their notebooks while they are solving the questions.
- Teacher walks around the classroom and observe the students.
- Teacher answers students' questions while students are trying to solve the worksheets.
- Teacher gives the answers and wants students to check their answers.
- Teacher asks students which questions they slog.
- Teacher solves students' questions on the board by asking students each step.

Transition: "Now I want you to write your reflections about summation notation in three minutes."

E. Evaluate (4 minutes)

- Teacher wants students to write a small journal in three minutes. In this journal the teacher wants students to write about which difficulties they came across in summation notation topic and in which questions they slog on the worksheet.

9. Closure & Relevance for Future Learning

- Teacher wants students to remember the properties of summation notation and by asking them the teacher writes properties which they

remember on the board and if necessary the teacher reminds them the properties of summation notation [**Knowledge**]

- Teacher distributes the second worksheets as homework and wants students to do and bring the worksheet the next lesson.

10. Modifications

- If students cannot remember the formulas of the summation notation the teacher will give some clue and remind them.
- If students cannot understand any question the teacher will want help from other students in the classroom.
- If the students cannot remember the functions the teacher will remind them the necessary information to solve the questions.
- If the students cannot remember the trigonometry the teacher will remind them the necessary information to solve the questions.

Worksheet
Summation Notation

1. Express each of the following by the summation notations and find each sums.

(a) $1+5+9+13+17+\dots+81$

(b) $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{79}{80!}$

2. Find the unknowns for each of the following equations.

(a)

$$\sum_{i=1}^n (i^2 + i) = 40$$

(b)

$$\sum_{i=1}^3 \left(\sum_{n=1}^4 (n^2 - i) \right)$$

4. $f(x) = 2x + 3$ and $x_k = 2k - 1$ are given. Find,

$$\sum_{k=1}^3 x_k = 40$$

(b)

$$\sum_{k=1}^5 (2k^3 - k^2) = A$$

3. Complete the following sums.

(a)

$$\sum_{i=1}^3 \left(\sum_{n=1}^4 (n^2 - i) \right)$$

5. Show that

$$\sum_{k=1}^n \frac{1}{2k-1} - \sum_{k=1}^n \frac{1}{2k} = \sum_{k=n+1}^{2n} \frac{1}{k}$$

6.

$$\sum_{k=-1}^{29} (-1)^{k+1} \cdot (2k+3) = ?$$

Homework
Summation Notation

1. Express each of the following by the summation notation and find each sum.

(a) $13+15+17+\dots+63$

(d) $\frac{1}{7.8} + \frac{1}{8.9} + \frac{1}{9.10} + \dots + \frac{1}{34.35}$

(b) $3^3 + 4^3 + 5^3 + \dots + 10^3$

(e) $\frac{1}{2^4} + \frac{1}{2^5} + \dots + \frac{1}{2^{13}}$

(c) $5.6+6.7+7.8+\dots+14.15$

2. Find the unknowns for each of the following questions.

(a)

$$\sum_{k=0}^7 (k - a) = 40$$

(c)

$$\sum_{i=1}^3 \sin \frac{nk}{6} = \frac{3 + \sqrt{3}}{n}$$

(b)

$$\sum_{n=-1}^6 (an + 3) = 30$$

(d)

$$\sum_{i=1}^2 \left(\sum_{k=1}^2 ki + k + i \right) = x$$

3. Compute the following sums.

(a)

$$\sum_{i=1}^3 \left(\sum_{j=1}^2 (i, j) \right)$$

(b)

$$\sum_{k=1}^7 \left[\sum_{n=1}^3 (k^2n + n) \right]$$

4. $x^2 + 7x + 14 = 0$ has the roots x_1 and x_2 .

Compute the sum of $\sum_{k=1}^2 x_k^3$

5. Find the remainder when $\sum_{k=1}^{100} k!$ is divided by 6.

6. $f(x) = 3x + 5$ and $x_1 = 3, x_2 = 7$ are given. Find;

Find $\sum_{k=1}^{100} k!$

7. $\sum_{k=1}^{12} k \cdot 2^{-k} = ?$