Date: 12.12.2013
Teacher: Çiğdem Özdemir
Number of Students: 17
Grade Level: 9
Time Frame: 45

Mathematics Learning Plan: Algebraic Word Problem Review

1. Goal(s)

- To develop an understanding of factorization of quadratics by applying them to the word problems.

2A. Specific Objectives (measurable)

- Students will be able to express word problems as quadratic equations.
- Students will be able to practice quadratic equations through factorization.
- Students will recall their algebra skills with solving algebraic word problems.
- Students will review algebraic expressions.

2B. Ministry of National Education (MoNE) Objectives

- İkinci dereceden bir bilinmeyenli denklemleri çözer.
- $ax^2 + bx + c$ biçimindeki cebirsel ifadelerin; tam kare ve iki kare farkına ait özdeşlikler de kullanılarak çarpanlara ayrılmasıyla ilgili uygulamalar yapılır.

2C. NCTM-CCSS-IB or IGCSE Standards:

- All students should understand the meaning of equivalent forms of expressions, equations, inequalities, and relations; (NCTM)
- All students should use symbolic algebra to represent and explain mathematical relationships (NCTM)
- Students will be able to factorize quadratic equations with common factor, difference of squares, trinomial, four term expressions. (IGCSE)

3. Rationale

- Students will understand that using algebra is essential for solving word problems.
- Students will relate algebra to the real life with solving word problems such as area and age problems.

4. Materials

- Computer,
- Overhead projector,
- Power Point presentation including the word problems which are going to be solved at the lesson.
- Two different color board markers.
- Worksheet

5. Resources

- Pearson, International Mathematics for the Middle Years 5.
- Oxford, International Mathematics for Cambridge IGCSE (0607)

6. Getting Ready for the Lesson (Preparation Information)

- Teacher prepares name cards for students and writes students' name on those cards before lesson.
- Teacher prepares a checklist including student names.
- Teacher prepares a Power Point presentation including the word problems which are supposed to be solved in the class.
- Teacher prepares the attached worksheet

7. Prior Background Knowledge (Prerequisite Skills)

- Students should have known solving quadratic equations by factorizing perfect squares, difference of squares and factorizing with common factor.
- Students should have known the distribution rule in algebraic expressions.

Lesson Procedures

Transition: "As you learned how to solve quadratic equations by factorizing at previous lessons, today we are going to review algebraic expressions by solving word problems about quadratic equations. Firstly, let's make a proof with using your algebra skills."

8A. Engage (5)

- Teacher asks students if they knew that 2=1, and asks them if they could make a proof of this equality.
- Teacher gets some comments from students and then starts to prove that 2=1

  Assume that, x=y

  1) Multiply both sides by x $x^2 = xy$

2) Subtract y^2 from both sides $x^2 - y^2 = xy - y^2$

3) Factorize both sides $(x - y)(x + y) = y(x - y)$

4) Divide by (x-y), $x + y = y$

5) Now, if we let x=y=1, then $2=1$

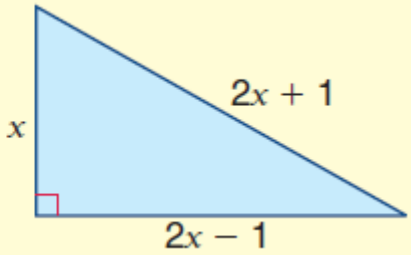
- Teacher makes the proof on the board with asking questions at each step such as "What do I get when I multiply both sides by x" at step1, "What was the expansion of difference of squares?" at step 2, "What should I do at the next step?" after step3.
- Teacher doesn't tell the fallacy at this part, as she will return to this proof while solving the word problem at the explanation part.
- After completing the proof, teacher asks students if they were convinced by this proof and gets comments of the students.

Transition: "Now it is time to apply your algebra skills into the word problems. If you can express a word problem algebraically, you can easily solve problems by using your algebra skills."

B. Explore (10)

- Teacher reflects two word problems about quadratic equations via PowerPoint presentation and wants students to write those questions in their notebooks with a title "Algebraic Word Problem Review". The problems are below.

A right-angled triangle is drawn so that the hypotenuse is twice the shortest side plus 1 cm, and the other side is twice the shortest side less 1 cm. Find the length of the hypotenuse.



Jenny is y^2 years old and her daughter Allyson is y years old. If Jenny lives to the age of 13y, Allyson will be y^2 years old. How old is Allyson now? (*Note:* the difference in ages must remain constant.)

- Teacher gives time students to think about the problems.
- Teacher walks around in the class, observes student work and takes notes on the checklist about students' problem solving process.
- Teacher asks students probing questions while students are thinking about questions. For example for second problem, teacher doesn't give the "Note" and asks if the age difference between two people changes in different years and asks "why" after student responses. Then teacher gives an example with comparing her own age and a student's age in different years.

Transition: Now let's solve the problem together. Please do not write the solution while we are solving together. I will give time to write."

C. Explain (10)

- Teacher asks students what they understood from the problem and writes down on the board.
- Teacher asks some students about their solutions as she observed while students were working on the problems.
- Teacher asks students why they thought in that way.
- Teacher solves the problem in different ways of different students.
- Teacher solves the problems in her own way on the board
- At the first question, teacher doesn't give the triangle with the question and expects students to place values on a right angled triangle.
- Teacher asks "What is the Pythagoras Theorem?" and writes the Pythagoras theorem on the board.
- Teacher asks "What do we get if we apply the Pythagoras theorem into this triangle"
- Teacher asks "What was the perfect square expansion?"
- For second problem, teacher asks "What can y be in order to make sense?"
- When teacher comes to the step at second problem"$y^2 = 7y$" , asks students what to do now.
- In this point, teacher returns to the proof of "2=1" and explains the fallacy showing;

$$(x - y)(x + y) = y(x - y)$$
$$x + y = y$$

and says "We cannot simplify (x-y), as we chose at the beginning x=y, so x-y=0 and we cannot divide an expression to 0." Then teacher returns back to the problem and says "We cannot simplify y, because y might be zero"

Transition: "Now I am distributing a worksheet. Solve the questions in the worksheet individually and ask me if you have questions."

D. Extend (10)

- Teacher distributes the attached worksheet and wants students to work individually.

- While students are working, teacher walks around in the class, takes notes on the checklist and discusses with students about the problems individually.

Transition:"Now let's see what we have learned today"

E. Evaluate (5)

- Teacher wants students to write down some sentences in their notebooks about what they learned during this lesson.
- After students write down some sentences in their notebooks, teacher gets comments from students about what they have learned from this lesson, at which kind of problems students had difficulties, etc.

9. Closure & Relevance for Future Learning

- At the end of the lesson, teacher makes a brief summary of the lesson.
- Teacher gives homework from the "Oxford, International Mathematics for Cambridge IGCSE (0607)", word problems in page 124.

10. Specific Key Questions:

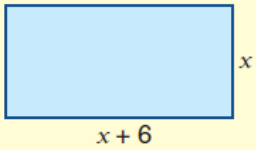
- How can you prove that 2=1? (Analysis)
- What was the expansion of differences of squares? (Knowledge)
- What is the Pythagoras theorem? (Knowledge)
- What do we get if we apply the Pythagoras theorem into this triangle (Application)
- What did you understand from this problem? (at each problem) (Analysis)
- How do you design a plan of this problem? (at each problem) (Synthesis)
- How can you express this problem algebraically? (at each problem) (Synthesis)
- What can y be in order to make sense?" (at the second problem) (Analysis)
- What is the common factor of this expression? (at the solution of the problems) (Application)
- How can I expand this expression? (at perfect square or difference of squares expressions) (Application)
- What we learned today? (Evaluation)

11. Modifications

- If students cannot remember factorizing quadratics with common factors, trinomials, difference of squares or four term expressions, teacher makes a brief recall of factorization of quadratics.
- If some students can understand the fallacy, teacher explains it to the whole class.

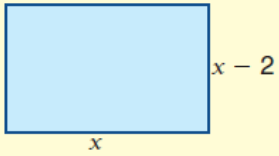
Algebraic Word Problem Review Worksheet

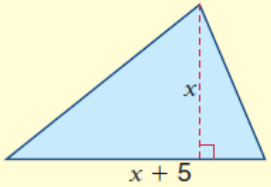
1)

a 

Find the dimensions of this rectangle if the length is 6 cm longer than the breadth and its area is 40 cm^2.

b The width of a rectangular room is 2 metres shorter than its length. If the area of the room is 255 m^2, find the dimensions of the room.



c 

The base of a triangle is 5 cm longer than its height. If the area of the triangle is 7 cm^2, find the length of the base.

2)

a Michelle threw a ball vertically upwards, with its height h, in metres, after a time of t seconds, being given by the formula:

$$h = 8t - t^2$$

Find after what time the ball is first at a height of 12 m.

b The sum, S, of the first n positive integers is given by the formula

$$S = \frac{n}{2}(n+1)$$

Find the number of positive integers needed to give a total of 78.

c For the formula $s = ut + \frac{1}{2}at^2$, find the values of t if:

i $s = 18, \quad u = 7, \quad a = 2$

ii $s = 6, \quad u = 11, \quad a = 4$

iii $s = 7, \quad u = 1, \quad a = 6$

3)

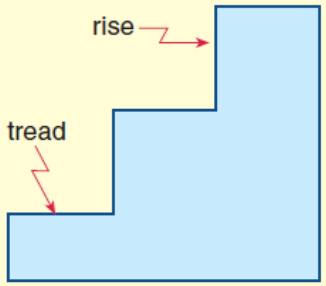
An n-sided polygon has $\frac{1}{2}n(n-3)$ diagonals. How many sides has a figure if it has 90 diagonals?

4)

Kylie bought an item for \$$x$ and sold it for \$10.56. If Kylie incurred a loss of x per cent, find x.

5)

The rise and tread of a staircase have been connected using the formula $r = \frac{1}{2}(24 - t)$, or $r = 66/t$ where r and t are measured in inches. (One inch is about 2·54 centimetres.)



a If the tread should not be less than 9 inches, what can be said about the rise?

b Graph both functions on the same set of axes and compare the information they provide.

c What are the points of intersection of the two graphs? Check the accuracy of your graphs by solving the two simultaneous equations,

$$\begin{cases} r = \frac{1}{2}(24 - t) & \ldots \ (1) \\ r = 66/t & \ldots \ (2) \end{cases}$$

(*Hint:* Substitute (2) into (1) and solve the resulting quadratic equation.)

d Convert formulae (1) and (2) to formulae applicable to centimetres rather than inches.

e Do the measurements of staircases you have experienced fit these formulae? Investigate other methods used by builders to determine r and t.