Date: 17.04.2014
Teacher: Çiğdem Özdemir
Number of Students: 15
Grade Level: 11 IBHL-TM
Time Frame: 45 minutes

Mathematics Learning Plan: Matrices

1. Goal(s)

- Students will be able to develop an understanding about the concept of matrices.

2A. Specific Objectives (measurable)

- Students will be able to define a matrix, determine its dimension and find the intended row, column and element.
- Students will be able to differentiate a row matrix, a column matrix, a zero matrix, a square matrix, a diagonal matrix, a triangular matrix and express the identity matrix.
- Students will be able to add two matrices and identify the properties of addition of matrices.
- Students will be able to multiply a matrix by a scalar and identify its properties.
- Students will be able to multiply two matrices and identify the properties of multiplication operation.
- Students will be able to find the transpose of a matrix and identify its properties.

2B. Ministry of National Education (MoNE) Objectives

- Matrisi örneklerle açıklar, verilen bir matrisin türünü belirtir ve istenilen satırı, sütunu ve elemanı gösterir.
- Kare matrisi, sıfır matrisini, birim matrisi, köşegen matrisi, alt üçgen matrisi ve üst üçgen matrisi açıklar, iki matrisin eşitliğini ifade eder.
- Matrislerde toplama işlemini yapar, bir matrisin toplama işlemine göre tersini belirtir, toplama işleminin özelliklerini gösterir ve iki matrisin farkını bulur.
- Bir matrisi bir gerçek sayı ile çarpma işlemini yapar ve özelliklerini gösterir.
- Matrislerde çarpma işlemini yapar ve çarpma işleminin özelliklerini gösterir.
- Bir matrisin devriğini (transpozunu) bulur ve özelliklerini gösterir.

3. Rationale

- The topic of matrices is useful for organizing data, and solving systems of linear equations.
- In fact, it is crucial for the systematic study of discrete problems in all branches such as computer science or industrial engineering.
- Especially for gamers with fast moving 3D graphics, the graphics card (the GPU) in a computer does nothing but billions of matrix calculations per second to render 3D objects nicely on a 2-dimensional surface (the screen)
- Moreover, matrix topic is an enjoyable topic and you wouldn't want to do any mistakes on the national exam in this topic.

4. Materials

- Two colors of board markers
- PowerPoint presentation including the definitions of specific matrices and properties of matrix operations**.**
- Worksheet-1 and Worksheet-2 print-outs.

5. Resources

- TED College Textbook for 11th grades.
- http://www.mathsisfun.com/algebra/matrix-multiplying.html
- MoNE Textbook for 11th grades.
- http://www.shelovesmath.com/algebra/advanced-algebra/matrices-and-solving-systems-with-matrices/

6. Getting Ready for the Lesson (Preparation Information)

- Teacher prepares name cards for students and writes students' name on those cards before the lesson.
- Teacher prepares a checklist including student names.
- Teacher prepares a PowerPoint presentation including the definitions of specific matrices and properties of matrix operations**.**
- Teacher prepares Worksheet-1 and Worksheet-2

7. Prior Background Knowledge (Prerequisite Skills)

- Students should have basic algebra knowledge.

Lesson Procedures

Transition: Good morning everyone. We are going to conduct the first period together. Although I remember most of your names, I will distribute your name cards one more time, please put them on your desk. I hope you remember my name as well. Today we are going to start a new topic "matrices"

8A. Engage (5 min.)

- *T*he teacher asks students whether they know how to demonstrate/write a matrix.
- Then she writes

$$A=\begin{bmatrix}9 & 7\\ 6 & 8\end{bmatrix}$$

and says "this is a matrix" and asks "why do you think we show numbers or variables in the array form? Why do you think it is useful? Then, teacher creates a table including these numbers:

	Rock	Classic
Male	9	7
Female	6	8

- Thus, the teacher emphasizes that matrices are useful for organizing data.
- The teacher gives extra information about the rationale of the topic.
- The teacher says that we generally show matrix with capital letters. Then teacher asks how many rows and columns does this matrix have. Accordingly, she writes 2*2 at the sub of the matrix and says “this is dimension of the matrix”
- Teacher writes

B=[4] and asks “Is this a matrix as well?” “How many rows and columns does this matrix have? So she writes the dimension of the matrix B as 1*1.

- Teacher picks the number “6” from matrix A and asks the location of the matrix in terms of its row and column. Thus, she writes a_{22}=6. Accordingly, she asks students a_{11}, a_{12} and a_{21}.
- Thus, teacher makes students write the general form of a matrix which is,

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}_{m \times n}$$

Transition: Teacher distributes the Worksheet-1 and says “There are some special types of matrices that you will need to know while solving questions. Now, try to match these specific types of matrices with their names”

B. Explore (10 min.)

Worksheet-1

$$A = \left[a_{ij} \right]_{mxn} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \square & a_{1n} \\ a_{21} & a_{22} & \square & \square & a_{2n} \\ a_{31} & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ a_{m1} & \square & \square & \square & a_{mn} \end{bmatrix}_{mxn}$$

Match the specific types of matrices with their names.

1)row matrix …
2)column matrix …
3)zero matrix …
4)square matrix …
5)diagonal matrix …
6)identity matrix …
7)scalar matrix …
8)triangular matrix …

$$G = [-1\ 3\ 5\ 2\ -3]$$

$$C = \begin{bmatrix} 6 \\ 72 \\ 4 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 3 & 6 & 9 \\ 5 & -8 & 10 \\ 1 & 7 & 2 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 6 & 0 \\ 3 & 5 & 8 \end{bmatrix}$$

$$H = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -2/7 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Fill in the blanks according to your matchings above.

If a matrix is a row matrix…………
If a matrix is a column matrix………..
If a matrix is a zero matrix………..
If a matrix is a square matrix……...
If a matrix is a diagonal matrix…………
If a matrix is a identity matrix………..
If a matrix is a scalar matrix………..
If a matrix is a triangular matrix……...

Transition: "Now please open your notebooks and take the notes of the definitions and properties on your notebook"

C. Explain (15 min.)

Equivalent matrices: Two matrices A and B are equal if they have the same dimensions and corresponding elements are equal.

(The teacher writes two matrices on the board and asks if the two matrices are equal or not.)

* Are A and B equal? (Yes, they are)

$$A=\begin{bmatrix} 4 & \cos(0) \\ \sin(0) & -2 \end{bmatrix}$$

$$B=\begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix}$$

Addition of Matrices: Two matrices A and B can be added if they have the same dimension by adding corresponding elements. The result is also a matrix in the same dimension.

* Find A+B

$$A=\begin{bmatrix} 10 & 13 \\ -11 & -8 \end{bmatrix}$$

$$B=\begin{bmatrix} -9 & -13 \\ 11 & 9 \end{bmatrix}$$

* Find A+ B. (The teacher expects the students to realize that they cannot add two matrices which have different dimensions)

$$A=\begin{bmatrix} 10 & 13 \\ -11 & -8 \end{bmatrix}$$

$$B=\begin{bmatrix} 2 & 6 & 3 \\ 2 & 1 & 0 \end{bmatrix}$$

Multiplication of a matrix by a scalar: If k $\in\mathbb{R}$ and A is any matrix, then kA is the new matrix for which each entry of A has been multiplied by k. The result is a matrix having the same dimension as A.

* Find -½ A, where

$$A=\begin{bmatrix}10 & 2\\ -8 & 0\end{bmatrix}$$

PROPERTIES

1) A+B=B+A (commutative property)
2) A+(B+C)=(A+B)+C (associative property)
3) A+0=0+A=A (identity of addition)
4) A+(-A)=(-A+A)=0 (additive inverse)
5) k(A+B)=kA+kB (distributive property)
6) (k + c) A= kA+cA (distributive property)
7) (k+A)=k.I+A, where k $\in\mathbb{R}$ and A is a square matrix.

ASK the students if we need to write the same set of rules for subtraction? WHY?

Transpose Matrix: In the matrix A, if corresponding rows and columns are interchanged then it is called "transpose matrix of A", denoted by A^T.

$$A=[a_{ij}]_{m\times n}\Rightarrow A^T=[a_{ji}]_{n\times m}$$

(Teacher writes an example of a transpose matrix)

* $A=\begin{bmatrix}2 & 6 & 3\\ 2 & 1 & 0\end{bmatrix}$ then find A^T.

$$A^T=\begin{bmatrix}2 & 2\\ 6 & 1\\ 3 & 0\end{bmatrix}$$

Properties:

$$(A+B)^T=A^T+B^T$$

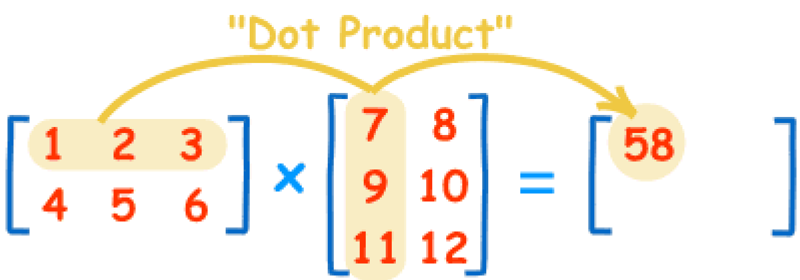
$$((A)^T)^T=A$$

$$(k.A)^T=k.A^T$$

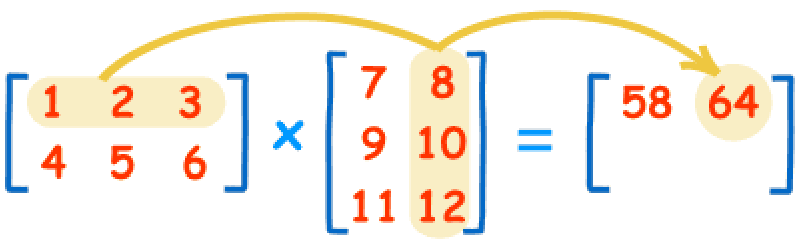
Multiplication of Matrices: Matrix multiplication has a special definition. The first step is to take the first row of left hand matrix and and pair it up with the first column of the right hand matrix. Multiply the pairs and sum of the products. The product gives a single number. This is the element of the first row and the first column of the resulting matrix. This calculation is continued as first row to the second column of the second matrix, and so on.

Matrices can be multiplied, if the number of columns in the first matrix equals to the number of rows in the second matrix.

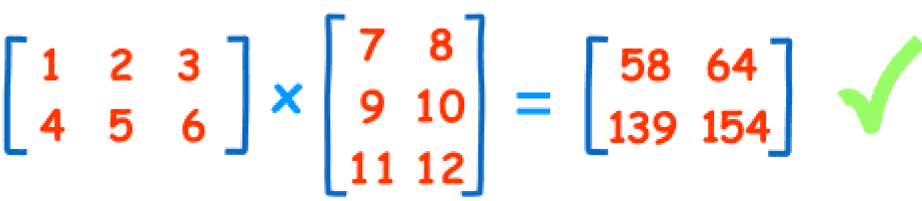
(Teacher gives an example of multiplication of two matrices)



$(1, 2, 3) \bullet (7, 9, 11) = 1\times7 + 2\times9 + 3\times11 = 58$



$(1, 2, 3) \bullet (8, 10, 12) = 1\times8 + 2\times10 + 3\times12 = 64$



Transition: Teacher distributes the worksheet below, and wants students to work on the worksheet with their pair.

D. Extend (10 min.)

Worksheet-2

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1)

$A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \\ 6 & 7 \end{bmatrix}$ is a matrix. Hence, find $a_{21} + 3a_{22} - a_{31}$.

2)
$A=(a_{ij})_{3\times 2}$
$A=\begin{bmatrix} 1 & 1/2 \\ -3/2 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} a-3 & 1 \\ -3 & a-1 \end{bmatrix}$ are given. If 2A=B then find a.

3)
$A=\begin{bmatrix} 2 & -1 \\ 5 & 3 \\ 4 & 7 \end{bmatrix}$ and $B=\begin{bmatrix} -5 & 4 & 0 & 9 \\ 6 & -3 & 2 & 11 \end{bmatrix}$ are given. Hence, find $A.B$

4)
$A=\begin{bmatrix} 3 & 1 & -6 \\ 3 & 0.5 & 1 \\ 3 & 8 & 0 \end{bmatrix}$ is given. Find $(kA)^T$

5) Assume that we want to find the final grades for 3 students and we know what their averages are for tests, projects, homework, and quizzes. We also know that tests are 40% of the grade, projects 15%, homework 25%, and quizzes 20%. Here's our data:

Student	Tests	Project	Homework	Quizzes
Ayşe	92	100	89	80
Burcu	72	85	80	75
Can	88	78	85	92

Perform a matrix multiplication to find the final grades of Ayşe, Burcu and Can.

E. Evaluate (5 min)

- Teacher evaluates students' performances during the exploration process by keeping a checklist.

9. Closure & Relevance for Future Learning

- The teacher asks the students to tell what they have learned in this lesson and notes them on the board. She informs the students that their teacher would continue in the second period. The teacher thanks the students.
  10. Specific Key Questions:

Why do you think we show numbers or variables in an array form? Why do you think it is useful?

- Are these two matrices equal?

Is this a matrix as well?

Can I add these two matrices?

Do we need to write the same set of rules for subtraction? Why?

Can I multiply these two matrices?

What is the dimension of the transpose of a 2x3 matrix?

11. Modifications

If no time left, Worksheet-2 can be given as homework.