Date: 24.03.2014
Teacher: Çiğdem Özdemir
Number of Students: 22
Grade Level: 11 Regular
Time Frame: 70

Mathematics Learning Plan: Sequences and Series

1. Goal(s)

- Students will understand the concept of sequences and series by applying them to real life problems.

2A. Specific Objectives (measurable)

- Students will be able to determine if a list of numbers refers a sequence or not.
- Students will be able to construct sequences through given information.
- Students will be able to find the general term of a given sequence.
- Students will be able to do operations with sequences
- Students will be able to find the general terms of arithmetic and geometric sequences.

2B. Ministry of National Education (MoNE) Objectives

- Dizi, sonlu dizi, sabit dizi kavramlarını ve dizilerin eşitliğini açıklar.
- Genel terimi veya indirgeme bağıntısı verilen bir sayı dizisinin terimlerini hesaplar.

2C. NCTM-CCSS-IB or IGCSE Standards:

- Students should generalize patterns using explicitly defined and recursively defined functions.
- Students will recognize and use simple arithmetic and geometric sequences.

3. Rationale

- Students will see that there are a lot of sequences in the nature and their daily lives by solving real life problems.
- This lesson will enable students to define the sequences and see their pattern when they encounter a sequence in their daily lives.

4. Materials

- Computer,
- Overhead projector,

- Power Point presentation including the properties of sequences and the chess board story.
- Worksheets

5. Resources
- Oxford, IB Diploma Programme, Mathematics Standard Level
- Hease Mathematics, Mathematics for the International Student, High Level
- MEB, Ortaöğretim Matematik 11. Sınıf Ders Kitabı

6. Getting Ready for the Lesson (Preparation Information)
- Teacher prepares name cards for students and writes students' name on those cards before lesson.
- Teacher prepares a checklist including student names.
- Teacher prepares a Power Point presentation including the properties of sequences and the chess board story.
- Teacher prepares the attached worksheets.

7. Prior Background Knowledge (Prerequisite Skills)
- Students should know making proof through induction.
- Students should know sum notation.

Lesson Procedures

Transition: Teacher introduces herself and distributes students' name cards that she prepared before in order to get to know students well. Then teacher says "Today we are going to start a new topic: sequences and series. Before I start the lesson, I want to tell you a story about chess game"

8A. Engage (5)
- Teacher reflects a chess game image on the board and tells the story of the chess game:
  "In ancient times, there was a tyrant king named King Shihram. One day one of his subjects, Sissa Ibn Dahir wanted to teach him how important all of his people were. He invented the game of chess for the king, and the king was greatly impressed. The king insisted on Sissa Ibn Dahir choosing his own reward. The wise man Sissa Ibn Dahir asked for one grain of rice for the first square, two grains for the second square, four grains of rice for the third square, and so on, doubling the rice on each successive square on the board.
  The king laughed at first and accepted as it seemed to him so little grain on the first few squares. By halfway he was surprised at the amount of grain being paid, and soon he realized his great error: he owed more grain than there was in the world."

- After telling the story teacher asks students each amount of grain of each square and writes down on the board as 1,2,4,8,16,…

Transition: "Now let's see if this number list refers a sequence"

B. Explore (5)

- Teacher asks students "What expression gives the number of grains of rice for the nth square?"
- Teacher asks "If I say that this number list refers a sequence what can you tell me about the sequences?" By this way, teacher expects students to express the properties of a sequence in their own words.
- Then teacher asks if they can find the next three term of 2,3,5,… sequence. She expects different responses such as 7,11,13 and 8,12,17. Here she emphasizes that if you have a reason of your pattern, it refers a sequence.

Transition: Now let's see how we demonstrate sequences.

C. Explain (30)

- Teacher explains the terminology of sequences. Then she distributes the attached worksheet.

- Teacher assigns the first three questions from the worksheet to solve.

1) Write down the first four terms of the sequence if you start with:

   a 4 and add 9 each time　　**b** 45 and subtract 6 each time

2) Describe the following number patterns and write down the next 3 terms:

   a 1, 4, 9, 16, ...　　**b** 1, 8, 27, 64, ...　　**c** 2, 6, 12, 20, ...

3) Write down the terms of the sequence $(a_n) = (2n - 1)$.

- After solving these questions, teacher asks the students to look for patterns or differences in the examples seen so far, expecting they recognize ideas like adding or multiplying by the same number.
- Then teacher explains that finding general terms helps finding next terms of the sequences.
- Teacher introduces the ways of finding the general term of sequences with using recursive and explicit definitions. Teacher gives Fibonacci Numbers as an example for recursive formula. Then she assigns the question below and gives time students to solve the question.

A sequence is defined by $u_1 = 3,\ u_n = u_{n-1} - 4,\ n > 1.$ Find:

a u_2 **b** u_3 **c** u_4 **d** u_5

- Teacher represents the general term of a sequence as a function.
- By demonstrating the general term as a function, teacher asks students the domain and range of this function.
- Teacher assigns problems including operations of sequences from the worksheet, so students could find out how to do operations with sequences

Consider the two sequences:

$$(a_n) = \left(\frac{1}{n}\right) \qquad (b_n) = \left(\frac{n+1}{n^2}\right)$$

Find:

$(a_n) + (b_n)$ $(a_n) - (b_n)$ $(a_n).(b_n)$ $(a_n):(b_n)$

- Teacher shows the constant sequences and equal sequences, and asks the question below from the worksheet
  If the sequence $(a_n) = \left(\frac{3n-k}{n+2}\right)$ is a constant sequence, find k.
- Teacher gives some examples of sequences and wants students to find out what an arithmetic sequence is. Then she wants students to

find the general term formula of an arithmetic sequence by looking these sequences.

Arithmetic Sequences

Consider the sequences below.

$(a_n) = (n+3) = (4, 5, 6, 7, \ldots\ldots, n+3, \ldots\ldots)$, $(b_n) = (3n+2) = (5, 8, 11, 14, \ldots\ldots, 3n+2, \ldots\ldots)$

$(c_n) = (2n+1) = (3, 5, 7, 9, \ldots\ldots, 2n+1, \ldots\ldots)$, $(d_n) = (2^n) = (2, 4, 8, 16, \ldots\ldots, 2^n, \ldots\ldots)$

Find the differences between each successive term.
Find the sequence which is different than others.

- After students explore the formula, teacher gives the definition of arithmetic sequence:

  $(\mathbf{a}_n)$ is arithmetic if and only if $\mathbf{a}_{n+1} - \mathbf{a}_n = d$

  for all positive integers n
  where d is a constant called the **common difference**.

- Teacher gives the examples of geometric sequences and expects students to find out the general formula of a geometric sequence.
  "Consider the sequences below

$$(a_n) = (3^n) = (3, 9, 27, 81, \ldots, 3^n, \ldots),\qquad (b_n) = \left(\left(\frac{2}{3}\right)^n\right) = \left(\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \ldots, \left(\frac{2}{3}\right)^n, \ldots\right)$$

$$(c_n) = \left(4.\left(\frac{1}{3}\right)^{n-1}\right) = \left(4, \frac{4}{3}, \frac{4}{9}, \frac{4}{27}, \ldots, 4.\left(\frac{1}{3}\right)^{n-1}, \ldots\right),\qquad (d_n) = \left(\frac{1}{n+1}\right) = \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots, \frac{1}{n+1}, \ldots\right)$$

Find the ratios between each successive term.
Find the sequence which is different than others."

- After students explore the formula, teacher gives the definition of geometric sequence:

$(\mathbf{a}_n)$ is geometric if and only if $\frac{\mathbf{a}_{n+1}}{\mathbf{a}_n} = \boldsymbol{r}$

for all positive integers n
where $\boldsymbol{r}$ is a constant called the ***common ratio***.

Transition: "Now solve the questions in the worksheet in 15 minutes.

D. Extend (15)

- Teacher distributes a worksheet including extended problems, and gives 15 minutes to solve them.

Transition: Now let's see what we have learnt

E. Evaluate (10)

- Teacher creates a chart on the board and fills it according to student's responses.

Sequences

Arithmetic Sequence	**Geometric Sequence**

9. Closure & Relevance for Future Learning

- Teacher wants students to write their weaknesses and strengths related to this topic in a few words at the end of the lesson.

10. Specific Key Questions:

- What expression gives the number of grains of rice for the nth square? (Analysis)
- If I say that this number list refers a sequence what can you tell me about the sequences? (Comprehension)
- What are the next three terms of the sequence 2,3,5…? What is your reason? (Analysis)
- Can you see any patterns or differences from the examples seen so far? (Analysis)
- What kind of definition of general term is this? (Knowledge)
- As general term of a sequence is a function, what is domain and range of this function? (Application)
- Which values can I give for n and which values can I get for a_n? (Comprehension)

- What kind of sequence is this? (By writing $(\mathrm{a}_n) = (3,3,3,3, \ldots ,3, \ldots))$ (Comprehension)
- What is the general term of this sequence? (Application)
- After students solve sixth question from worksheet 1, teacher asks “What if I substitute 5 and 8 for n and equalize them?” (Comprehension)
- If three of these four sequences are arithmetic sequence, which one do you think is not an arithmetic sequence? (Analysis)
- Why this sequence is different from others? (Analysis)
- What is the property of arithmetic sequence? (Synthesis)
- By showing the formula of arithmetic sequence, teacher asks “Does this formula correspond your finding?” (Evaluation)
- If three of these four sequences are geometric sequence, which one do you think is not a geometric sequence? (Analysis)
- Why this sequence is different from others? (Analysis)
- What is the property of geometric sequence? (Synthesis)
- By showing the formula of geometric sequence, teacher asks “Does this formula correspond your finding?” (Evaluation)

11. Modifications
- If students cannot find the general term of the chess board sequence, teacher may give some hints. On the other hand, if it comes to students very easy, teacher may expect students to add the terms of the sequence. Also if students cannot find the different sequence among arithmetic and geometric sequences, teacher may give hints such as making them to look at the difference and the ratio between two successive terms of the sequences.

Slide 1

Slide 2

$(a_n) = (a_1, a_2, \ldots\ldots, a_n, \ldots.)$

or

$\{u_n\} = \{u_1, u_2, \ldots., u_n, \ldots.\}$

Slide 3

(a_n) is arithmetic if and only if $a_{n+1} - a_n = d$

for all positive integers n
where d is a constant called the **common difference**.

Slide 4

(a_n) is geometric if and only if $\frac{a_{n+1}}{a_n} = r$

for all positive integers n
where r is a constant called the ***common ratio***.

Slide 5

Sequences and Series

Arithmetic		Geometric	
Sequence	Series	Sequence	Series

Worksheet 1

1) Write down the first four terms of the sequence if you start with:

a 4 and add 9 each time

b 45 and subtract 6 each time

2) Describe the following number patterns and write down the next 3 terms:

a 1, 4, 9, 16, ...   b 1, 8, 27, 64, ...   c 2, 6, 12, 20, ...

3) Write down the terms of the sequence $(a_n) = (2n - 1)$.

4) A sequence is defined by $u_1 = 3$, $u_n = u_{n-1} - 4$, $n > 1$. Find:

a u_2 b u_3 c u_4 d u_5

5) Consider the two sequences:

$$(a_n) = \left(\frac{1}{n}\right) \qquad (b_n) = \left(\frac{n+1}{n^2}\right)$$

Find:

$(a_n) + (b_n)$ $(a_n) - (b_n)$ $(a_n).(b_n)$ $(a_n):(b_n)$

6) If the sequence $(a_n) = \left(\frac{3n-k}{n+2}\right)$ is a constant sequence, find k.

Arithmetic Sequences

Consider the sequences below.

$(a_n) = (n + 3) = (4, 5, 6, 7, \ldots\ldots, n + 3, \ldots)$, $(b_n) = (3n + 2) = (5, 8, 11, 14, \ldots\ldots, 3n + 2, \ldots)$

$(c_n) = (2n + 1) = (3, 5, 7, 9, \ldots\ldots, 2n + 1, \ldots)$, $(d_n) = (2^n) = (2, 4, 8, 16, \ldots\ldots, 2^n, \ldots)$

Find the differences between each successive term.
Find the sequence which is different than others.

7) Find the 28th term of a sequence if its first term is 7 and the common difference is -2.

8) In a finite sequence, if the first term is a_1, the last term is a_n and the common difference is d, Find how many terms this sequence has.

Geometric Sequences

Consider the sequences below:

$(a_n) = (3^n) = (3, 9, 27, 81, \ldots, 3^n, \ldots)$,

$(b_n) = \left(\left(\frac{2}{3}\right)^n\right) = \left(\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \ldots, \left(\frac{2}{3}\right)^n, \ldots\right)$

$(c_n) = \left(4.\left(\frac{1}{3}\right)^{n-1}\right) = \left(4, \frac{4}{3}, \frac{4}{9}, \frac{4}{27}, \ldots, 4.\left(\frac{1}{3}\right)^{n-1}, \ldots\right)$,

$(d_n) = \left(\frac{1}{n+1}\right) = \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots, \frac{1}{n+1}, \ldots\right)$

Find the ratios between each successive term.

Find the sequence which is different than others.

9) Find the general term of the sequence which has the first term1 and common ratio 4.

10) "(a_n) is a geometric sequence and r is the common ratio of this sequence. Hence find the general term of this sequence in terms of the first term and the common ratio.

Worksheet 2

1) Find the next two terms of:

a 95, 91, 87, 83, ... **b** 5, 20, 80, 320, ... **c** 1, 16, 81, 256, ...

2) Evaluate the first *five* terms of the sequence $(15 - (-2)^n)$.

3) A sequence is defined by $u_1 = 1, \quad u_n = \dfrac{1}{1 + u_{n-1}}, \quad n > 1$.

a Find the next four terms of the sequence.

b Display the first 5 terms on a graph.

4) Valéria joins a social networking website. After 1 week she has 34 online friends. At the end of 2 weeks she has 41 friends, after 3 weeks she has 48 friends, and after 4 weeks she has 55 friends.

a Show that Valéria's number of friends forms an arithmetic sequence.

b Assuming the pattern continues, find the number of online friends Valéria will have after 12 weeks.

c After how many weeks will Valéria have 150 online friends?

5) Identify the following sequences as arithmetic, geometric, or neither:

a $7, -1, -9, -17, ...$

b $9, 9, 9, 9, ...$

c $4, -2, 1, -\frac{1}{2}, ...$

d $1, 1, 2, 3, 5, 8, ...$

e the set of all multiples of 4 in ascending order.

6) Find k if $3k$, $k-2$, and $k+7$ are consecutive terms of an arithmetic sequence.

7) Find k given that 4, k, and $k^2 - 1$ are consecutive terms of a geometric sequence.

8) Find, in simplest form, a formula for the general term u_n of:

a 86, 83, 80, 77, ... **b** $\frac{3}{4}$, 1, $\frac{7}{6}$, $\frac{9}{7}$, ... **c** 100, 90, 81, 72.9, ...

Hint: One of these sequences is neither arithmetic nor geometric.